THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

330:542 **Information Theory and Coding** **Spring 1997**

Takehome Exam

You have ALL WEEK to answer the following questions. You are NOT allowed to collaborate, but any textbooks you find are fair game. By the way, I never know whether questions are easy or hard, so pay no attention to the order. Furthermore, some questions may have more than one answer or no answer. So budget your time accordingly.

GOOD LUCK!

1. (*50 points*) I've warned you about multivariate functions and the dangers of checking for convexity with partial derivatives. Well, in this problem you're going to derive conditions on partial derivatives which guarantee convexity.

   Convexity of a function $f(\mathbf{x})$ is implied if

   $$f(\lambda \mathbf{x}_1) + (1-\lambda)\mathbf{x}_2) \leq \lambda f(\mathbf{x}_1) + (1-\lambda) f(\mathbf{x}_2)$$

   We also know that convexity of a single-variable function is guaranteed if it's second derivative is positive everywhere.

   Using the provided definition as a guide, derive the conditions on the second partials of $f()$ which guarantee that $f()$ is convex. HINT: Consider $f()$ as a function of the single variable λ.

2. (*50 points*) Derive conditions (if possible) for which a hidden markov process is itself markov. That is, suppose we have a markov process X with states numbered 1 through N and we partition the states to form a process Y with "states" numbered 1 through $K < N$.

   Let $P_{i|j}$ be the transition probabililty matrix for process X, $i, j = 1, 2, ..., N$.

   (a) (*25 points*) For any given partition, what condition must $P_{i|j}$ satisfy so that Y is markov?

   (b) (*25 points*) What general condition on $P_{i|j}$ allows any partition of X states to be markov?

3. (*50 points*) Find the entropy rates of the following processes:

   (a) (*25 points*) A Markov process with states $i = 0, 1, ...$ and transition probabilities $P_{i|i+1} = .5$, $P_{i|i-1} = .25$ and $P_{i|i} = .25$ for $i! = 0$ and for $i = 0$ $P_{0|1} = .5$ and $P_{0|0} = .5$.

   (b) (*25 points*) A partition process Y on the previous markov process X where "state" j in Y is composed of the k consecutive X states $k * j, k * j + 1, ..., k * j + k - 1$ with $j = 0, 1, ...$

4. (*50 points*)

   (a) (*25 points*) On page 49 of you text are three paragraphs under historical notes. Considering only the characters (ignore punctuation marks, numerals), estimate the "transition probability" between letters in English. That is, compile a relative frequency measure of how probably character e follows character y and so on. From your measurements estimate the entropy rate of written English. Give reasons why your calculation might be exact. You should include the "space" character in addition to the standard 26 letter alphabet.

   (b) (*25 points*) Repeat the first part for a passage of similar length in your mother tongue. What is the entropy rate of the written form of your mother tongue. If your mother tongue is English then repeat the first part considering pairs of letters followed by pairs of letters.