THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

330:542 **Information Theory and Coding** **Spring 1996**

Quiz 1

You have 80 minutes to answer the following 3 questions. Please read the exam in its entirety before beginning work. Please underline and/or box your final answers for each part to aid in the grading process. You are allowed your one side of an 8.5X11 standard sheet of note paper, a pencil and a blue book. YOU MUST SHOW ALL WORK TO RECEIVE CREDIT.

The problems may be hard or easy. I can never tell ahead of time.

GOOD LUCK!

1. *(40 points)* **Source Coding:**

   A particle moves according to a random walk on the integers. At time t a particle at position k will move to $k+1$ or $k-1$ with probability p and $1-p$ respectively. Assume the position of the particle at $t=0$ is $k=0$.

   (a) *(10 points)* What is the entropy of the particle at $t = 0, 1, 2$? What is the entropy of the particle at $t = \infty$.

   (b) *(10 points)* If we wish to be kept informed of the position of the particle at each time step, what average information rate in bits per time step is necessary? Please compare this to the entropy rate of this Markov process. Evaluate for $p = 0.5$.

   (c) *(10 points)* Say we wish to be kept informed of the particle location at $t = 2n$, $n = 1, 2, ...$ What is the necessary average information rate in bits per time step. Please compare this to the entropy rate of this process. Evaluate for $p = 0.5$.

   (d) *(10 points)* Now suppose we wish to find the particle at random times t_i where the $\Delta_i = t_i - t_{i-1}$ are independent geometrically distributed random variables. Please derive an EXPRESSION for the average number of bits necessary PER TIME STEP to resolve the particle location at the desired times.

   HINT: Write down your answer first in terms of the relevant sequences of random variables, then use the weak law of large numbers. Do not look to simplify too much.

2. *(30 points)* **Distribution Optimization:**

   This problem is based on a problem which arose in last proof of chapter 5 in Cover and Thomas. Don't worry if you don't see the relationship. You won't need it to solve the problem.

Assume we have distributions of the form

$$p_i = \gamma(i)2^{-i} / \sum_{j=1}^{\infty} \gamma(j)2^{-j}$$

where $\gamma(i)$ is an indicator function which takes on values zero or one as a function of i. We define the entropy of the distribution as

$$H(\mathbf{p}) = \sum_{i=1}^{\infty} p_i \log \frac{1}{p_i}$$

Please find the indicator function $\gamma(j)$ which maximizes this entropy. Or if you prefer, show that $H(\mathbf{p}) \leq 2$.

3. *(30 points)* **Coding Objectives:**

   You are given an objective function,

   $$\Gamma(\mathcal{C}, p(), F()) = \sum_x p(x)F(\ell_{\mathcal{C}}(x))$$

   on a code $\mathcal{C}$ with codeword lengths $\ell_{\mathcal{C}}(x)$ where $F()$ is some function. For mean codeword length we have $F(x) = x$ and the Huffman procedure finds binary codes which minimize $\Gamma()$.

   Using your detailed knowledge of the proof of Huffman optimality, please identify restrictions on the class of functions $F()$ for which the Huffman procedure finds a code which minimizes $\Gamma()$ for ANY $p(x)$.