Rutgers University
The State University Of New Jersey
College of Engineering
Department of Electrical and Computer Engineering

330:542 | Information Theory and Coding | Spring 1996

Problem Set 3
ISSUED: 2/14/96

1. C&T problem 3.3
2. C&T problem 4.6
3. C&T problem 4.14
4. C&T problem 5.11
5. C&T problem 5.14
6. A source has an alphabet of 4 letters. The probabilities of the letters and two possible sets of binary code words for the source are given below:

Letter	Prob	Code-I	Code-II
a_1	0.4	1	1
a_2	0.3	01	10
a_3	0.2	001	100
a_4	0.1	000	1000

For each code, answer the following questions (no proofs or numerical answers are required).

(a) Does the code satisfy the prefix condition?

(b) Is the code uniquely decodable?

(c) What is the mutual information provided about the event that the source letter is a_1 by the event that the first letter of the code word is 1?

(d) What is the average mutual information provided about the source letter by the specification of the first letter of the code word? Give a heuristic description of the purpose of the first letter in the code words of code II.

7. Let $\{0,1\}$ be the set of all finite length binary sequences. Let $C = \{c_i | l \leq i \leq N\}$ be a finite set of codewords, i.e., C is contained in $\{0,1\}$. Assume C is prefix free (i.e. satisfies the prefix condition). Define

$$g(C) \;=\; \sum_{i=1}^{N} 2^{-|c_i|}$$

where $|c_i|$ is the length of codeword c_i. The Kraft inequality is a statement that $g(C) \leq 1$. C is said to be *complete* if adding any new codeword to C produces a new set C' which is not prefix free.

(a) Prove that C is complete **iff** $g(C) = 1$

(b) Now suppose that C has infinitely many codewords. Prove that if $G(C) = 1$ then C is complete. Be careful here. Notice the **iff** is gone!

(c) C is said to be *full* if every binary sequence in $\{0,1\}$ is either in C, is a prefix of some sequence in C or has a prefix in C. This holds if C is finite or infinite. Prove that C is full **iff** C is complete.

(d) Find an example of a complete infinite set of codewords C such that $g(C) < 1$. Can you find a set of codewords such that $g(C)$ is arbitrarily small and C is still complete? (i.e., show that the converse to part 7b is false).