Rutgers University
The State University Of New Jersey
College of Engineering
Department of Electrical and Computer Engineering

330:542 | Information Theory and Coding | Spring 1996

Problem Set 2

ISSUED: 1/31/96 | SOLUTIONS: NEXT WEEK

1. C&T problem 2.12
2. C&T problem 2.15
3. C&T problem 2.29
4. Convexity and all that:
   (a) You are given a scalar function $f(x, y)$ which is convex in the pair of its arguments (x, y). Show that $f(r(z), s(z))$ is convex in z if both $r(z)$ and $s(z)$ are linear functions of z.
   (b) Suppose $f(x, y)$ is both convex and non-decreasing in both x and y. Show that $f(r(z), s(z))$ is convex in z if both $r(z)$ and $s(z)$ are convex functions of z.
   (c) Suppose $f(x, y)$ is both convex and non-increasing in both x and y. Show that $f(r(z), s(z))$ is convex in z if both $r(z)$ and $s(z)$ are concave functions of z.
   (d) Is $f(\mathbf{x})$ convex in $\mathbf{x}$ if $\frac{\partial^2 f}{\partial {x_i}^2} \geq 0$ for $i = 0, 1, 2...N$? Prove or disprove.
   (e) Extend your proof of part (a) to include scalar functions of vector arguments $f(\mathbf{x}, \mathbf{y})$ and vector functions of vector arguments $\mathbf{r}(\mathbf{z})$ and $\mathbf{s}(\mathbf{z})$. A linear vector function, $\mathbf{q}(\mathbf{z})$ of a vector $\mathbf{z}$ is defined as, $q_i(\mathbf{z}) = \sum_{j=1}^{N} a_{ij} z_j + constant$.
5. Entropy and all that:

   A switching node has traffic matrix t_{ij} where t_{ij} is the amount of traffic to be routed from input port i to output port j. For any M-input N-output switching node, an $M \times N$ traffic matrix $\mathbf{T} = \{t_{ij}\}$ may be defined. Let the "complexity" of the switching node be defined as

   $$\Pi(\mathbf{T}) = -\sum_{i=1}^{M}\sum_{j=1}^{N} t_{ij} \log(t_{ij}/r_i) - \sum_{i=1}^{M}\sum_{j=1}^{N} t_{ij} \log(t_{ij}/c_j)$$

   where $r_i = \sum_{j=1}^{N} t_{ij}$ (row sum) and $c_j = \sum_{i=1}^{M} t_{ij}$ (column sum). Note: An $M \times N$ matrix has M rows and N columns. Assume $\sum_{ij} t_{ij} = 1$.

   (a) Provide a lower bound for the complexity. What set $\{t_{ij}\}$ attains this lower bound?
   (b) Provide an upper bound for the complexity. What $\{t_{ij}\}$ attain this upper bound?

   Suppose two switches are cascaded. Specifically, let the first switch have M inputs and Q outputs. Let the second switch have Q inputs and N outputs. The overall result is an $M \times N$ switch. We can represent the traffic through the combination as a 3-dimensional matrix $\mathbf{D} = \{d_{ijk}\}$ where d_{ijk} is the amount of traffic that emanates from input $i = 1, 2, ...M$, passes through intermediate output $j = 1, 2, ...Q$ and leaves the switch at output $k = 1, 2, ...N$. This representation allows us to define three matrices $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{T}$ where

   $$\mathbf{X} = \{x_{ij} = \sum_{k} d_{ijk}\}$$

is the traffic matrix of the first switching stage,

$$\mathbf{Y} = \{y_{jk} = \sum_i d_{ijk}\}$$

is the traffic matrix of the second stage and

$$\mathbf{T} = \{t_{ik} = \sum_j d_{ijk}\}$$

is the overall traffic matrix from input to output.

(c) Prove that $\Pi(\mathbf{X}) + \Pi(\mathbf{Y}) >= \Pi(\mathbf{T})$; i.e., the overall switching complexity is less than or equal to the sum of the intermediate switching node complexities. You may assume that $\sum_{ijk} d_{ijk} = 1$.